To sum up, changes in reliability and maintainability can be measured by their effects on the maintenance float; these effects are quite pronounced, are fairly linear in the ranges normally used, and can be expressed both as technical float factors and in money terms.

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Stability Derivatives by Rheoelectric Analog

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Introduction

THE use of the electrical analog to solve aerodynamic problems is well known. The electrical analog method for determining apparent masses was suggested in the Journal of the Royal Aeronautical Society in April 1965. However, with the method used at Norair, integration in the complex plane is not required. The required answer can be accomplished basically with but one simple measurement of resistance. The method can also be extended to use an electrolytic tank to obtain apparent masses for arbitrary three-dimensional shapes.

Northrop Norair has been working for a number of months on a NASA contract to study the aerodynamics of lifting reentry bodies. This study includes investigation of both static and dynamic coefficients over a wide range of Mach numbers. Its main purpose is to develop estimating methods applicable to lifting bodies in general; results are being applied to the NASA M-2 and HL-10 vehicles.

Slender body theory seemed promising for application to this program. The basic idea of slender body theory is that each body cross section can be studied independently of other cross sections, and the aerodynamic force contribution of each can be summed over the body length to obtain aerodynamic stability derivatives (see Fig. 1). The important parameter

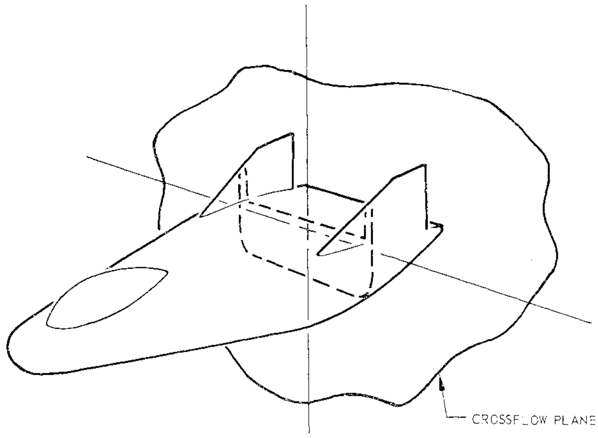

Fig. 1 Slender body theory.

of each cross section which allows computation of these derivatives is the apparent mass.

In the case of a single body moving through an otherwise unbounded and undisturbed infinite fluid, it can be shown that the entire effect of the fluid may be represented by the addition of apparent masses to the inertia of the solid. It is desirable, therefore, to determine simplified methods for evaluating apparent masses for fluid flow.

Application of Electrical Analog

In two-dimensional flow, a relationship between apparent mass and the complex function describing the flow can be found. The relationship between the apparent mass and the flow function shows that the determination of the residue of the flow function will allow calculation of apparent mass. However, determination of this residue is not mathematically simple for arbitrary shapes. Therefore, some simple measurement, such as using an electrical analog technique, may be useful. Following this clue led to the method described here, which utilizes measurements of the apparent resistance presented by an arbitrary shape cut out of a sheet of electrically conducting paper.

To illustrate this concept, consider the following. For two-dimensional flow, the relationship between apparent mass and the flow function for an arbitrary shape is

$$\begin{aligned} A_{12} - i(A_{11} + \rho s) &= \rho \oint w_1 dz \\ A_{22} + \rho s - iA_{12} &= \rho \oint w_2 dz \end{aligned} \tag{1}$$

(see Ref. 1), where w_1 is the complex potential for a unit flow in the x direction, and w_2 is for unit flow along y. Also, A_{ij} is the apparent mass in the i direction due to flow in the j direction, and $\oint wdz$ is $2\pi i$ times the residue of the flow function.

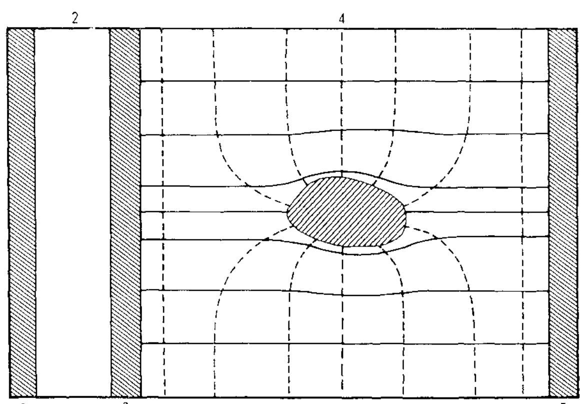

Fig. 2 Residue evaluation.

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Table 1

Path	$\int\phi dx + i\int\phi dy$	$i\int\psi dx + \int\psi dy$
Bottom	$\int\phi dx$	0
Right side	iVW	$-\int\psi dy$
Top	$\int\phi dx$	$-irIL$
Left side	0	$-\int\psi dy$

The apparent mass coefficients are then

$$m_{11} = A_{11}/\rho s = (1/s)I\oint w_1 dz - 1$$
$$M_{22} = A_{22}/\rho s = (1/s)I\oint w_2 dz - 1 \qquad (2)$$

where I means "imaginary part of."

Now consider the field ditribution of a sheet of electrically conducting paper of size L by W with a body shape cut out and an infinitely conducting strip at each end, as diagramed in Fig. 2. The function of the second strip on the left is to form a calibration region. Applying the well-known principle of the electrical analog, let ϕ (the voltage, or the velocity potential) be zero at the left end, and have the value V at the right end. Let ψ (the current lines, or streamlines) be zero at the bottom, and have the value rI at the top, r being the resistivity of the paper and I the total current flow.

Now

$$\oint wdz = \oint(\phi + i\psi)(dx + idy) \qquad (3)$$

and the integral can be evaluated around the edge of the paper as shown in Table 1. Therefore, the imaginary part of the integral is

$$I\oint wdz = VW - iIL \qquad (4)$$

and, since $V = \Omega I$, where Ω is the total measured resistance of the paper,

$$I\oint wdx = VW - (rVL/\Omega) = VW[1 - (rL/w\Omega)] \qquad (5)$$

But rL/W is Ω_0, the resistance of the paper before the hole or body was cut out. Also, to obtain unit freestream velocity, let $V = L$. The integral now becomes

$$I\oint wdx = LW[(\Omega - \Omega_0)/\Omega] \qquad (6)$$

$$m_{11} = A_{11}/\rho s = (LW/\rho s)[(\Omega - \Omega_0)/\Omega] - 1 \qquad (7)$$

Similarly,

$$m_{22} = A_{22}/\rho s = (LW/s)[(\Omega - \Omega_0)/\Omega] - 1 \qquad (8)$$

where the body is now turned 90° from the previous case.

This result indicates that apparent masses for arbitrary cross sections can be measured by an engineer at his desk using data obtained with a sheet of conducting paper and an ohmeter. These can then be used to calculate slender body stability derivatives.

This result also suggests the possibility of determining apparent masses of arbitrary three-dimensional shapes using an electrolytic tank and a dielectric model.

Method of Taking Measurements

The method just described was put to practical application at Northrop. Initially, simple geometric forms were used for which the theoretical solution is known. Thus, the errors connected with empirical evaluation of apparent mass could be established. Also, details of the technique of measurements and the tools used for this purpose could be improved. The Wheatstone bridge circuitry used at Northrop has a certified accuracy of about 1 in 2000.

The first simple forms were ellipses. They were cut in the paper with their longer axis both normal to and parallel to the flow. The Teledeltos paper used is 8.5 × 11.7 in. As noted in Fig. 2, the sheet is divided into five parts. Parts 1, 2, and 5 are each about ½ in. wide and are covered with electrically conducting silver paint that can be obtained commercially. Part 2, about 1.7 in. wide, is the calibration strip. Part 4 is the test section.

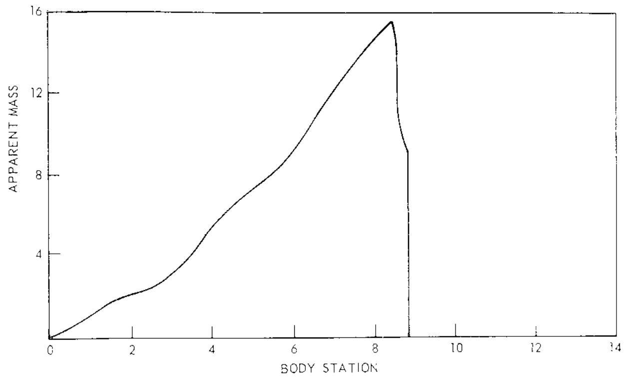

Fig. 3 M-2 apparent mass distribution.

Measurements began with calibration of the paper. First, the resistance Ω_c of the calibration strip (part 2 in Fig. 2) was measured, then the resistance Ω_0 of the blank test section (part 4).

Experience gained with a large number of such measurements on different sizes of paper and under different environmental conditions has demonstrated that the ratio

$$K_0 = \Omega_0/\Omega_c \qquad (9)$$

remains constant for a reasonably long period of time.

Next, an elliptical hole was cut in the paper, with the center of the ellipse on the center of the test section. The value K was again measured and the following quantity determined:

$$(\Omega - \Omega_0)/\Omega = (K - K_0)/K \qquad (10)$$

Test results showed that good estimates can be obtained by this method.

The cutout shape should be about 5% of the total area of the conducting sheet between the two painted stripes. If the resistance measurement is made with an accuracy of 1 in 1000, the answer can have an accuracy of approximately 2%.

If the body is painted on with conducting paint instead of being cut out, the apparent mass for flow about the body rotated 90° will be obtained by the same procedure.

The method just described was applied to actual lifting bodies. Figure 3 shows the variation along body axis of apparent mass for vertical flow for a typical lifting body.

By integration of graphs similar to Fig. 3, derivatives of forces and moments such as $C_{L\alpha}$, $C_{y\beta}$, $C_{m\alpha}$, $C_{m_q+\dot{\alpha}}$, etc., were found for incompressible flow. These were in good agreement with wind-tunnel test data. Methods were also developed to extend the subsonic value through transonic and into the supersonic regime, resulting in fair agreement with experiments.

Conclusions

1) The simple measurement of over-all resistance of a sheet of conducting paper with arbitrary shapes cut out can be used to determine apparent mass with good accuracy.

2) The electrical analogy method for determining apparent masses is a useful but simple tool for obtaining static and dynamic aerodynamic stability derivatives.

3) The technique warrents further work to develop wider application, such as determinations of three-dimensional apparent masses utilizing an electrolytic tank.

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